

# ISOGEOMETRIC ANALYSIS: PROGRESS AND CHALLENGES

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## ABSTRACT

Geometry is the foundation of analysis yet modern methods of computational geometry have until recently had very little impact on computational mechanics. The reason may be that the Finite Element Method (FEM), as we know it today, was developed in the 1950's and 1960's, before the advent and widespread use of Computer Aided Design (CAD) programs, which occurred in the 1970's and 1980's. Many difficulties encountered with FEM emanate from its approximate, polynomial based geometry, such as, for example, mesh generation, mesh refinement, sliding contact, flows about aerodynamic shapes, buckling of thin shells, etc. It would seem that it is time to look at more powerful descriptions of geometry to provide a new basis for computational mechanics.

The purpose of this talk is to explore the new generation of computational mechanics procedures based on modern developments in computational geometry. The emphasis will be on Isogeometric Analysis in which basis functions generated from NURBS (Non-Uniform Rational B-Splines) and T-Splines are employed to construct an exact geometric model. For purposes of analysis, the basis is refined and/or its order elevated without changing the geometry or its parameterization. Analogues of finite element  $h$ - and  $p$ -refinement schemes are presented and a new, more efficient, higher-order concept,  $k$ -refinement, is described. Refinements are easily implemented and exact geometry is maintained at all levels without the necessity of subsequent communication with a CAD (Computer Aided Design) description.

In the context of structural mechanics, it is established that the basis functions are complete with respect to affine transformations, meaning that all rigid body motions and

constant strain states are exactly represented. Standard patch tests are likewise satisfied. Numerical examples exhibit optimal rates of convergence for linear elasticity problems and convergence to thin elastic shell solutions. Extraordinary accuracy is noted for  $k$ -refinement in structural vibrations and wave propagation calculations. Surprising robustness is also noted in fluid mechanics problems. It is argued that Isogeometric Analysis is a viable alternative to standard, polynomial-based, finite element analysis and possesses many advantages. In particular,  $k$ -refinement seems to offer a unique combination of attributes, that is, robustness and accuracy, not possessed by classical  $p$ -methods, and is applicable to models requiring smoother basis functions, such as, thin bending elements, and strain-gradient and phase-field theories. A new modeling paradigm for patient-specific simulation of cardiovascular fluid-structure interaction is described, and a précis of the status of current mathematical understanding is presented.